

**CORPORATE INCOME TAX EVASION AND
EFFICIENCY LOSS**

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**A THESIS SUBMITTED
FOR THE DEGREE OF MASTER OF
SOCIAL SCIENCES**

**DEPARTMENT OF ECONOMICS
NATIONAL UNIVERSITY OF SINGAPORE**

2009

ACKNOWLEDGEMENTS

On the completion of my thesis, I would like to express my deepest gratitude to all those people whose kindness and advice have made this work possible.

I am greatly indebted to my supervisor Dr Younghwan In, for his effective advice, illuminating instructions, valuable comments and constant encouragement.

I would like to express my heartfelt gratitude to the professors and teachers at the Department of Economics, who have helped me a lot in the past two years.

Most of all, my thanks would go to my beloved family for their considerations and confidence in me all through these years. I also owe my sincere gratitude to my friends and my fellow classmates who gave me support during the difficult times.

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SUMMARY

We set up a model to investigate corporate income tax evasion. The main difference between individual tax evasion and corporate tax evasion is that the owners and the managers of a corporation are usually separate from each other, thus they need to cooperate when deciding to evade, which suggests that corporate tax evasion is a kind of principal-agent problem. Managers need to be given enough incentive to evade through a compensation package depending on the firm's after-tax profit rather than before-tax profit. We find that the optimal compensation scheme has to be altered under evasion, which means an evading firm bears an extra cost of efficiency loss in corporate governance in addition to the cost of being detected and penalized compared to an individual, which we believe is an important factor in a firm's decision making process and for explaining the great difference in observed data.

1. Introduction

There is a long history of taxation. Since there existed the concept of nation, there was also the concept of taxation. Taxation is the important way a government earns incomes. It seems that paying taxes is a citizen's inherent obligation, while imposing taxes is a government's natural right. There is a long history of tax evasion as well. Since there was taxation, there was tax evasion. Although some people voluntarily pay their taxes no matter due to their strong sense of responsibility or the fear that not paying is illegal, most people are not willing to pay out part of what they earn and seeking the opportunity to shirk their tax liability, especially observing that some people are not being punished for evading taxes. Therefore, governments can not merely announce a tax system and expect to collect all the taxes they have planed to raise. Tax authorities have to make effort to monitor and administer taxation, which is not costless. Thus, it is beneficial to investigate the characteristics and determinants of tax evasion, so that governments are able to improve their tax systems and enhance the enforcement of taxation.

First of all, it is necessary to determine the extent of tax evasion; however, it is difficult to measure because of its illegal nature. On one hand, no one will tell that her actual income is larger than what she reports to be subject to taxation when filling a questionnaire for research. On the other hand, the complexity of the law and the tax system itself makes it ambiguous when determining whether an activity is illegal tax evasion or legally permissible tax avoidance. The most credible source of information about the extent of tax evasion comes from two programs conducted by the U.S.

Internal Revenue Service. The Taxpayer Compliance Measurement Program, done from 1963 until 1988, was a program of intensive audits on a certain group of taxpayers, combining with the information obtained from other studies about particular sources of income, based on which, the IRS estimated the “tax gap”. Later, the program stopped for complaints that such research was unfair; until recently, a modified version, the National Research Program was implemented to analyze tax returns from the 2001 tax year. After more than ten years, the IRS released their estimates of tax underreporting gap for 2001.

In the IRS’s report, the aggregate tax underreporting gap was broken down into individual income tax, employment tax, corporate income tax etc. Looking into the data provided in Slemrod (2007), there are some interesting findings. Individual income tax is further divided into underreported non-business income and underreported business income. Non-business income is underreported by only 4 percent, while the underreporting rate of business income is much higher at 43 percent. This is probably resulted from the fact that non-business income, such as wages and salaries, or interest and dividends are reported by the employers or the firms who pay them. There is less incentive for the reporters to hide the real information for those who receive the money. However, the individual business income is under everyone’s own control. Thus, the 43 percent basically reflects the extent of individual tax evasion. This considerably higher rate is also associated with the high underreporting rate of self-employment tax, which is estimated as 52 percent. For corporate income tax, firms are classified into large ones and small ones based on their assets. Large

corporations underreport about 14 percent of their incomes, while the rate of small corporations is slightly more than twice (29 percent); both are lower than that of individual income tax.

Observing these data, we would like to raise several questions: why there is such great difference in the underreporting rate among individual business income, small corporations and large corporations? How tax evasion decisions are made and what is the distinction between individual income tax and corporate income tax?

Intuitively, there are two possible explanations for the reality. One is that the extent of penalty may be different between individuals and corporations; the other is that the extent of auditing may also not be the same. The penalty varies from country to country. In most countries, it is mainly in the form of a fine payment, but for severe cases, the evaders may be given criminal conviction. For a company, normally it is the legal person and the finance manager who take responsibility for the evasion. But within a certain country, if it is stipulated that people involving in severe cases or repeated cases should be put into jail, usually it applies to both individuals and managements of a company. The other explanation is different auditing frequency. A corporation's profit exceeds a person's income a lot, so a same or even a lower underreporting rate of corporations perhaps means a much larger quantity which may cause the tax authority to take more investigation, thus making the large corporations harder to evade compared to individuals. Perceived this fact, some papers considered the case under a varying auditing frequency which is related to the reported amount. However in our work, we would like to show a third possible answer, thus we will

assume there is no distinction in these two factors. We assume a same punishment for individual tax evasion and corporate tax evasion; besides we treat the possible legal penalty and reputation loss as in the monetary term. We assume a detection probability which is independent of the reported taxable income. In another word, an identical tax system is supposed for these two kinds of tax evasion, but it will be shown that corporations still exhibit lower underreporting tendency. We believe it results from the organization form of modern corporations. Furthermore, it gives some vague edification: unlike what we usually think, it seems that corporations do not need monitoring as much as individuals do.

Beginning with the classic article of Allingham and Sandmo (1972), researches on individual tax evasion developed a lot; however, although corporations play a central role in the tax system, corporate tax evasion has not gain much attention. Most studies on corporate tax evasion are not separate from those of individual ones in the fundamental concept: there is just one individual who decides whether and how much to evade. This is true for private business, where the owner runs the business herself, including managing the tax reporting issues. However, it does not work in the exact same way for public corporations. In public-held corporations, owners and managers are usually apart from each other. Owners benefit from tax evasion by retaining a higher after-tax profit, so that they have an incentive to evade tax as long as the expected utility exceeds that of truthfully reporting. But a key point is that owners generally do not report themselves, in stead, they have managers of the firm to make these arrangements for them. Therefore, corporate tax evasion is not purely a problem

for one individual, but rather a more complicated issue which involves the interaction between firm owners and their managers. Managers usually do not have the same incentive as owners. Tax evasion will not benefit managers if managers are compensated according to firm's before-tax profitability. Furthermore, managers may not be willing to participate in evasion since evasion is illegal, which might induce penalty on them. Hence, if a firm determines to evade tax, it should firstly give managers enough incentive to participate in this activity; in another word, firms should compensate managers for bearing the risk of tax evasion. This suggests that corporate tax evasion should be analyzed under a principal-agent framework, which seeks to find ways to align managers' incentives with firm owners'.

The first theoretical model of corporate tax evasion is proposed in Chen and Chu (2005). They argued that the nature of illegality of evasion produces the incompleteness of the compensation contract, which will distort the effort of the manager and cause internal efficiency loss. We doubt that the contract designed in Chen and Chu's paper is not workable in reality, so in this paper, we make some twists to their model. We will introduce their designation later, but in our model, we just use a common simple contract, incorporating a "stock portion", which tries to capture the after-tax profitability so that the manager is rewarded not only for her effort in managing the firm's operations but also for the risk in participating in the evasion. There is similar result as found in Chen and Chu: in the presence of tax evasion, compensation contract must be changed from the second-best compensation scheme, which will induce efficiency loss. The argument implies that firms have two sources

of costs when evading tax. One is the risk of being caught, which is also considered in analyzing individual income tax evasion. The other one is the efficiency loss, which we believe is the critical factor in corporate tax evasion. It can be used to explain what is found from the empirical data. Because of one more consideration of tradeoffs, firms exhibit a much lower underreporting rate compared to individuals. An individual will evade taxes if the expected profit is higher from evasion than that from honest reporting. A corporation will evade only when the expected profit is higher enough to cover the internal efficiency loss.

We offer two models. The first one is a simple model where we assume the firm owner can observe the true profit produced by the manager. The owner has two choices, ordering the manager to truthfully report or allowing her to underreport. We will show later that whenever the owner allows evasion, the manager will always choose to evade. This assumption is made to eliminate the possibility for the manager to overstate her performance. However, this kind of events does happen in the real world. Owners usually do not know the sound status of their firms just like the tax authority. They obtain the information by examining the financial statements prepared by the managers, facing a problem that the managers might have the incentive to over-report the profit so that they can earn a higher salary. Thus, in the second model, we loosen the restriction: the owner cannot observe the true profit any more. She has to design some certain compensation contracts to induce the manager to underreport, truthfully report or over-report. We will show that this fact brings more efficiency loss to the firm when tax evasion is a consideration.

The remainder of the thesis is organized as follows. In section 2 we provide a literature review of tax evasion, focusing on corporate income tax evasion. In section 3 we set up the model. We will first discuss Chen and Chu's model, and then explain why we need to change the assumptions and what our model looks like. Section 4 presents the findings. Section 5 concludes.

2. Literature Review

The theoretical model of tax evasion was first provided by Allingham and Sandmo (1972). In that paper, the authors analyzed how a risk-averse individual make decision on whether and how much to evade taxes. Since underreporting will not necessarily be detected, tax declaration decision is like gambling. Evasion may give the taxpayer a higher pay-off, but it's at the risk of being caught and penalized, thus being worse off. Rational individual chooses how much to evade to maximize her expected utility. The paper showed that optimal tax evasion level depends on the degree of risk aversion, applied tax rate, penalty rate and the chance of getting caught.

This simple model has been extended in the following years in different dimensions. One is to incorporate labor supply decision so that income is no longer exogenously given, which brings interaction between labor supply decision and tax evasion decision. Another aspect is to consider repeated games between taxpayers and tax authorities, as people will update their knowledge and adjust their decisions with previous experiences. Besides, there are researches on analyzing the situation that the probability of audit is a function of reported income other than a constant rate. Some other economists take into account the influence of people's moral sense and the fairness of the tax system. (Andreoni *et al*, 1998; Sandmo 2005)

While there are a number of papers addressing individual tax evasion following Allingham and Sandmo, the study regarding corporate tax evasion is much less. Most of the models assume that the owner of the firm makes decision on profit declaration.

Although the issue discussed is about business tax evasion, it is actually similar to that of individuals. It is still a portfolio-selection problem, in which decision maker selects the optimal level of the risky asset (evaded income which saves tax but induces possible penalty). The main difference is that with the presence of tax evasion, the choice of production might also be affected by tax rate, penalty size and auditing frequency. Marrelli and Martina (1988) analyzed an oligopolistic market and showed that optimal amount of tax evasion depends on the collusiveness as well as the relative market shares. Virmani (1989) considered a more general case of a perfect competitive market and concluded that tax evasion may lead to inefficient production and thus imposing costs on the economy. A series of papers focused on the relationship between these two decisions of a monopolist. Kreutzer and Lee (1986) explored how profit tax can reduce monopoly distortion since successful tax evasion will have an effect on the output level which maximizes the firm's after-tax profit. They showed that the overstatement of production costs induces the firms to increase output. This opinion challenged the generally held point of view that profit tax is neutral, i.e. it does not influence a monopolist's profit maximizing level of output. Wang and Conant (1988) derived a contrary result to them that the monopolist's optimal level of output is separate from the tax evasion decision, which reinforced the conventional view. However, in Wang (1990), by endogenizing the probability of detection and the penalty rate, it was concluded that the separability between the two decisions no longer holds and the tax-evading monopolist may restrict output, even worsening monopoly distortion. Further discussions can be found in Yaniv (1996),

Lee (1998), Panteghini (2000), Goerke and Runkel (2005).

However, the assumption that when choosing to underreport the profit, a firm acts in the same way as an individual does is only reasonable with those small-sized and privately-held firms. In large public companies, tax reporting decision, like most producing or investment decisions, is usually made by CFO or the vice-president who is in charge of tax affairs instead of being directly determined by the owners. Because of the separation between ownership and management, the problem of corporate tax evasion is more complicated than individual ones. Slemrod (2004; 2005; 2007) pointed out that corporate tax evasion should be analyzed under a principal-agent framework – “to align the incentives of the decision makers and the shareholders, the corporation should tie the agent’s compensation to observable outcomes that affect after-tax corporate profitability” (2007).

There is paper investigating how manager’s own preference in evasion influence her decision for the firm, e.g. in David Joulfaian (2000), but it is not the relationship we are going to talk about. One of the earliest theoretical models to analyze the problem in a principal-agent way was provided by Chen and Chu (2005), which is also the fundamental model this paper relied on. The authors considered a game between a risk-neutral principal (owner of the firm) and a risk-averse agent (manager). They indicated that tax evasion is illegal and a contract based on illegal actions is not enforceable. Manager bears the risk of being caught and punished, so the contract should be able to compensate the manager not only her effort but also her risk in order to induce her to participate in evasion, i.e. a higher wage when evasion is detected and

penalized. Besides, knowing that the contract will not be honored by the court if evasion is detected, the risk needs to be compensated *ex ante*, which leads to the incompleteness of the contract in the sense that the manager's pay is not contingent on whether evasion is detected or not. This failure in efficiently sharing the risk of evasion by the owners and the manager creates distortion in the manager's incentive in exerting efforts. Their model implies that in addition to the tradeoff between expected gain in evasion and the risk of getting caught, there is another tradeoff between expected gain and efficiency loss for corporations. Therefore, even if the expected gain from evasion less the expected penalty is positive, a firm will not necessarily evade as efficiency loss might be extremely high. The authors also discussed other potential reasons that can incur internal inefficiency when evasion exists, e.g. agent's extortion and repeated interaction.

Crocker and Slemrod (2005) examined this issue by considering a contractual relationship between the shareholders of a firm and the CFO it hires. Under their assumptions, the before-tax income of the firm is exogenously given (not depending on the CFO's effort). The CFO's private information here is the extent of the legally permissible tax reductions x , which the shareholders only have the idea of the distribution density. Besides, the actual reduction in taxable income R is known to the shareholders. Thus, there exists tax evasion of $(R - x)$ which might be penalized upon detecting. The main results found are: tax evasion will be reduced when penalties – no matter levied on the firm or the CFO – increase; however, the penalty imposed on CFO directly is more effective in reducing tax evasion, which provides guidance for

tax authorities in their enforcement policies.

In Desai *et al* (2003), the authors investigated the interaction between corporate taxes and corporate governance using a different approach. They considered a game involving tax authorities, inside shareholders who have the ability to divert corporate value, and outside shareholders. Since attempting to avoid tax makes a firm's financial state opaque to outside investors, it becomes harder for outsiders to control insiders. Thus, tax evasion will worsen the corporate governance. It is shown that higher tax rate will worsen corporate governance as it increases the return of diverting by insiders, thus providing more incentive for insiders to divert. On the contrary, increase in tax enforcement helps to increase the difficulty of the insiders in diverting corporate value into their own pockets, which favors the interest of outside shareholders, and increases the market value of the firm, although the tax payment is higher under this situation.

Later on, in Desai and Dharmapala (2006), the authors introduced the principal-agent framework into the analysis. They argued that decisions about tax evasion and rent diversion are interdependent on each other. Incentive compensation aims to align the incentive of agents with the interests of the principals so that it should induce the reduction in rent diversion and the increase in tax avoidance. Thus, higher-powered incentive compensations should lead to more tax evasion. However, the paper showed that the effect is ambiguous because of the interaction between the two decisions.

3. The model

3.1 Chen and Chu's model

Before introducing our model, we discuss Chen and Chu's first. In their paper, a standard principal-agent model where a risk-neutral principal hires a risk-averse agent is presented. The manager exerts a certain level of effort e and then the firm will realize a profit y with a density function of the realization of y , $f(y|e)$; it is assumed that $f(y|e)$ first-order stochastically dominates $f(y|e')$ if $e > e'$, which means more efforts results in higher expected profit. The profit y is verifiable and observable to both parties while effort level is the private information of the manager¹. In the first stage, the owner and the manager sign a contract of the compensation for the manager, $w(y)$, which depends on the realized profit y . In the second stage, the manager chooses an effort level to maximize her utility. The owner designs the compensation contract in the first stage by solving the manager's optimization problem in the second stage, in order to maximize her after-tax profit. This is the well-known method to give the manager enough incentive so as to work in the interest of the owners.

However, if the owner intends to evade part of the firm's true profit, the situation becomes complicated. Given that tax evasion is illegal, the authors argued that the previous contract will no longer be supported by the court and thus no longer enforceable. Therefore, the manager has no incentive to evade tax since her

¹ These are standard assumptions in principal-agent analysis developed by B. Holmsdrom (1979).

compensation will depend on the reported profit r rather than the true profit y . This is because on one hand $w(r)$ is less than $w(y)$ when evasion is profitable and successful, while on the other hand if the owner refuses to pay according to y , the court won't enforce her to do that as officially the firm's profit is r , so that rational owner will always renege on the contract². Knowing this, the manager will not be willing to help the owner to underreport profit. Chen and Chu argued that the owner has to alter her contract to induce the manager to do that. The new contract consists of two parts: a wage contract $w(r)$ depending on the reported profit r and a so-called "service" contract which is $w(y) - w(r)$. In the first stage, the owner designs the contract and the manager accepts it. In the second stage, the manager selects an effort level and a certain profit y is realized. Observing this profit y , the owner chooses how much the manager should report to the tax authority, r . This underreporting activity is seemed as an extra "service" provided by the manager. Whenever she acts in this way, she will get $w(y) - w(r)$ in return. It was argued that this contract is enforceable because officially reporting r means that the manager does provide the service and deserves this payment, which will be supported by the court³. With this new contract, the

² By "officially the firm's profit is r " it means that since the company reports its profit to the tax authority as r , the court will just take r as what the manager produces and according to which she can claim her compensation. The true profit y is verifiable and observable to both the owners and the manager, but not to the public without any investigation, otherwise the tax authority will always be able to obtain the real information and no evasion is possible to exist. Furthermore, here it is assumed that only the tax authority will take investigation into whether the profit is underreported, while the court's work is just to enforce the contract. We will discuss this later.

³ Again, whether r is real profit or not is not the court's concern. The problem for the court is to rule on how much should be paid to the manager by the company according to their contract, taken r reported on the financial statement published as true profit the manager produces. Therefore, under the former contract form, only $w(r)$

manager will be compensated $w(r) + [w(y) - w(r)] = w(y)$ if she evades according to the owner's order, which is the same as being paid just $w(y)$ if she does not follow the owner, but rather truthfully reporting the profit, so that the manager is indifferent between evading and not evading, where as assumed she will opt to evade⁴. Since actually the manager is paid according to the true profit y in this case, evasion decision does not affect the optimal compensation contract and optimal effort level⁵.

Furthermore, there is potential penalty on the manager. Suppose that the manager will be penalized by $x(y - r)$ ⁶ if evasion is detected, so that paying $w(y)$ is still not enough for the manager to participate in evasion; she needs to be compensated more for the risk. Again, this compensation should be made in the same way as the service contract does for the same reason. Thus, the contract becomes $\{ w(r) ; w(y) - w(r) + \pi(y) \}$, where $\pi(y)$ is the risk premium. In order to induce the manager to participate in evasion, this $\pi(y)$ should satisfy that the manager will be

needed to be paid, while under the latter form, in addition to $w(r)$, the "service payment" should also be paid.

⁴ Here the potential penalty to the manager is not considered. It will be explained in the next paragraph. This is for the purpose of showing that even the manager is determined by law not liable for evasion, though it rarely happens, the compensation contract has to be in a special form. The decision of evading in case of indifference is to ensure the analysis to be meaningful. However, it sounds unreasonable. We suggest one possible explanation for why the manager will choose to take the illegal activity: firstly, the compensation is the same, there is no loss for the manager to perform illegally since it is assumed in this case that she can distinct herself from evasion; secondly, if she does not obey the owners, she might lose her job in the future.

⁵ On one hand, the manager's optimization problem is not changed. On the other hand, it is showed that the optimal level of evasion is independent of the real profit, thus the difference of the owner's problem between evasion case and no evasion is just an additional constant term, which will not affect the solutions.

⁶ x is a function of $(y-r)$, which means the penalty depends on the size of the evasion. We will explain the detailed requirements x function should satisfy later in our model.

indifferent between receiving $w(y)$ for sure, i.e. reporting truthfully, and evading with the risk of being caught. Suppose that there exists a probability of p to be audited by tax authority and once audited, evasion will be certainly detected, $\pi(y)$ will be like that to ensure the following equation hold:

$$pu[w(r) + w(y) - w(r) + \pi(y) - x(y - r)] + (1 - p)u[w(r) + w(y) - w(r) + \pi(y)] = u[w(y)]$$

where $u(\cdot)$ is the agent's utility function.

3.2 Our concerns

Nevertheless, we doubt whether the above-stated contract is workable in reality. A key point for the contract to properly work is that the service contract will be supported by the court, which guarantees the manager to be compensated no worse under evasion. Chen and Chu agree that generally the court will only look into whether the manager fulfills the requirements in the contract when being asked to enforce the service contract, but will not check whether that r is true profit, so tax evasion will not be uncovered because of this contract form. This argument promises the feasibility of the contract proposed. However, we do believe that in the real world, the separation of wage contract and service contract implies that there is manipulation in the profit report, thus must lead the court to suspect and take investigation into the corporation. With investigations, tax evasion will be found; because it is illegal, on one hand, the legitimacy of the contract no longer exists, which means that the manager can no longer expect to get the “service payment” through it, on the other hand, there is extra penalty waiting for her, which tells that this kind of contract apparently cannot give

the manager enough protection that it intends to provide. Knowing all these, the manager is better not to participate in the tax evasion. To solve the problem, we abandon this kind of contract and make some more reasonable assumptions in our models.

We consider a usual contract, which states that manager should be compensated according to the reported profit r . r is verifiable and observable to both parties in both models. But as explained before, when taking tax into consideration, the manager needs to be compensated according to after-tax profit in order to induce evasion - this can be realized through a fraction of “stocks”. Assume that after-tax profit is completely reflected in the dividends paid and/or in the appreciation of the stock price. In another word, by given the stock part, the manager possesses a proportion of after-tax profit as a component of total compensation. Finally, the manager is compensated a wage part and a stock part. There is some difference between the wage here and that used in the standard principal-agent model, i.e. the wage is based on how much profit reported instead of how much actually produced. Also, there is some difference in the meaning of the stock part. We consider only one factor which influences the value of the stocks, i.e. the after-tax profit, and we directly treat the increase in the value of the stocks as the manager’s income. This is to let any potential losses and risks related to tax evasion be compensated by the stocks given to the manager. Moreover, let the manager to decide whether and how much to evade. There is a trade off for the manager between receiving higher wages but lower gain in stocks with reporting more and receiving fewer wages but more income from stocks with

evading more. A rational manager will choose to balance the tradeoff, and because the choice is made by the manager herself, there is no conflict between the owner and the manager. No matter whether to evade or not, how much to evade, it is an outcome maximizing both the manager's utility and the owner's utility.

We are aiming to check how evasion decision brings efficiency loss, so we only focus on the situations where the firm earns a positive profit, i.e. the firm should pay some tax according to law. If the firm loses within a certain time horizon, no income tax needs to be paid, thus no tax evasion decision. In another word, we suppose a prerequisite that the true taxable income is greater than zero.

For simplicity, we assume linear contracts in both models. Manager will get a salary linear in the reported profit and a proportion of stocks. In model 1, we assume the principal can observe true before-tax profit, so that any over-report will be prevented. In model 2, we loosen the assumptions: the owner does not observe the exact true before-tax profit, providing the chance of over-report to the manager. Because in model 1, over-report will not happen, to make the problem simple, we just assume a fixed wage part plus stocks; while in model 2, a general wage function that consists of a fixed part and a part increasing in the reported profit is considered. There is a tradeoff for the manager: reporting higher profit means higher wage but lower income as a shareholder; reporting lower profit means less wage but more earnings in stocks. We discuss model 1 first.

3.3 Model 1

Suppose that before-tax profit the firm can generate depends on the manager's effort. Thus, higher effort means higher expected wage as well as more disutility caused by working harder. Besides, the manager's earnings are also affected by her tax reporting strategy if the firm allows evasion because evasion may leave the firm with more after-tax income, benefiting shareholders, including the manager.

The firm owner is assumed to be risk-neutral, while the manager exhibits constant absolute risk-averse preferences. Let her utility be expressed by the following exponential function:

$$u(w, a) = -e^{-\eta[w - \psi(a)]}$$

where w is the total monetary compensation. $\eta > 0$ is the coefficient of absolute risk aversion, and $\psi(a)$ is a measure of the effort cost, a for effort level. Let the effort cost function to be quadratic as $\psi(a) = \frac{1}{2}ca^2$.

Suppose the owners and the manager sign a linear contract in the following form:

$$w = b + s(1 - t)(y - b)$$

where b is the fixed salary the manager will be paid and y is the realized before-tax profit (revenue subtracting all the costs other than the manager's salary). Thus, $(y - b)$ is subject to be taxed⁷. $s \in (0, 1)$ is the proportion of stocks given to the

⁷ b should be small relative to y , however, it is a choice variable of the owners in the issue we are discussing, which strictly speaking, affects the before-tax profit. We cannot treat it as the same as other costs, because all the other costs, such as costs of raw materials, operation costs and other employees' payroll etc, are determined by the manager as a result of her effort in running the company, while b is out of the manager's control. The ultimate

manager. $t \in (0,1)$ is the applied tax rate.

Further assume that the before-tax profit is equal to manager's effort level plus a noise term:

$y = a + \varepsilon$, where $\varepsilon \sim N(0, \sigma^2)$ ⁸. But remember that our discussion is restricted within the region of a positive profit.

3.3.1 No evasion case

The principal maximizes her utility by choosing the fixed salary b and the stock proportion s . Meanwhile, through the compensation contract, she indirectly chooses an optimal effort level. Thus, although effort is the manager's decision, we include it as one of the owners' choice variables. The principal's problem is as follows:

$$\max_{a,b,s} E(1-s)(1-t)(y-b)$$

subject to

$$E(-e^{-\eta[b+s(1-t)(y-b)-\frac{1}{2}ca^2]}) \geq u(\bar{w}) \quad (\text{IR})$$

and

$$a \in \arg \max_a E(-e^{-\eta[b+s(1-t)(y-b)-\frac{1}{2}ca^2]}) \quad (\text{IC})$$

where $u(\bar{w})$ is the reservation utility level of the manager and \bar{w} denotes the

before-tax profit actually is $(y-b)$, where y is the result of manager's behavior, while b is the result of owners' behavior, which should be separated. The impact on before-tax profit produced by b cannot be included in y , thus b should not be eliminated from the expression of taxable profit even though it is relatively small to y .

⁸ All the assumptions follow the model in *Contract Theory, Chapter 4: Hidden action, moral Hazard*. (Bolton, Patrick and Dewatripont, Mathias, c2005).

minimum acceptable monetary equivalent of the manager's compensation contract.

Given the contract, i.e. taking b and s as given, the manager tries to maximize her expected utility.

$$\begin{aligned}
E(-e^{-\eta[b+s(1-t)(y-b)-\frac{1}{2}ca^2]}) &= E(-e^{-\eta[b+s(1-t)(a+\varepsilon-b)-\frac{1}{2}ca^2]}) \\
&= (-e^{-\eta[b+s(1-t)(a-b)-\frac{1}{2}ca^2]}) E(e^{-\eta s(1-t)\varepsilon}) \\
&= (-e^{-\eta[b+s(1-t)(a-b)-\frac{1}{2}ca^2]}) (e^{\frac{\eta^2 s^2 (1-t)^2 \sigma^2}{2}})^9 \\
&= (-e^{-\eta[b+s(1-t)(a-b)-\frac{1}{2}ca^2 - \frac{\eta s^2 (1-t)^2 \sigma^2}{2}]}) = (-e^{-\eta \hat{w}(a)})
\end{aligned}$$

The maximization problem of the manager thus becomes to

$$\max_a \hat{w}(a) = b + s(1-t)(a-b) - \frac{1}{2}ca^2 - \frac{\eta s^2 (1-t)^2 \sigma^2}{2}$$

The first order condition is

$$s(1-t) - ca = 0 \Rightarrow a = \frac{s(1-t)}{c}$$

This is how much effort the manager will exert for any given incentive s .

⁹ See *Contract Theory, Chapter 4: Hidden action, moral Hazard*. (Bolton, Patrick and Dewatripont, Mathias, c2005). When a random variable ε is normally distributed with zero mean and variance σ^2 , we have

$$E(e^{\gamma \varepsilon}) = e^{\gamma^2 \sigma^2 / 2} \text{ for any } \gamma.$$

$$\begin{aligned}
E(e^{\gamma \varepsilon}) &= \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{+\infty} e^{\gamma \varepsilon} e^{-\frac{\varepsilon^2}{2\sigma^2}} d\varepsilon = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{+\infty} e^{-\frac{\varepsilon^2 - 2\gamma\sigma^2 \varepsilon}{2\sigma^2}} d\varepsilon = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{+\infty} e^{-\frac{(\varepsilon - \gamma\sigma^2)^2 - \gamma^2 \sigma^4}{2\sigma^2}} d\varepsilon \\
&= e^{\gamma^2 \sigma^2 / 2} \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{+\infty} e^{-\frac{(\varepsilon - \gamma\sigma^2)^2}{2\sigma^2}} d\varepsilon = e^{\gamma^2 \sigma^2 / 2}
\end{aligned}$$

since $\frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{+\infty} e^{-\frac{(\varepsilon - \gamma\sigma^2)^2}{2\sigma^2}} d\varepsilon$ is the area under a normal distribution with mean $\gamma\sigma^2$ and variance σ^2 ,

which equals 1.

Substitute the expression of a into the principal's initial problem, it can be reduced to

$$\max_{b,s} (1-s)(1-t) \left(\frac{s(1-t)}{c} - b \right)$$

subject to

$$b + s(1-t) \left(\frac{s(1-t)}{c} - b \right) - \frac{s^2(1-t)^2}{2c} - \frac{\eta s^2(1-t)^2 \sigma^2}{2} = \bar{w}$$

Reorganize the constraint and we can get

$$b = \left(\frac{s^2(1-t)^2}{2c} - \frac{\eta s^2(1-t)^2 \sigma^2}{2} - \bar{w} \right) / (-(1-s+st))$$

Substitute it into the objective function and differentiate with respect to s , we obtain the first order condition for s :

$$\frac{(1-t)^2(1-2s)}{c} + (1-t)^3(1-c\eta\sigma^2) \frac{2s+ts^2-4s^2+2s^3-2ts^3}{2c(1-s+st)^2} + \frac{\bar{w}t(1-t)}{(1-s+st)^2} = 0 \quad (1)$$

Denote the solution to equation (1) as s° , and substitute it into the expression of b and get b° . We call (b°, s°) the second-best compensation scheme.

Let us check the existence of the solution. Denote

$$g = \frac{(1-t)^2(1-2s)}{c} + (1-t)^3(1-c\eta\sigma^2) \frac{2s+ts^2-4s^2+2s^3-2ts^3}{2c(1-s+st)^2} + \frac{\bar{w}t(1-t)}{(1-s+st)^2}$$

$$g(s=0) = \frac{(1-t)^2}{c} + \bar{w}t(1-t) > 0$$

$$g(s=1) = -\frac{(1-t)^2}{c} - \frac{(1-t)^3(1-c\eta\sigma^2)}{2ct} + \frac{\bar{w}t(1-t)}{t} = \frac{1-t}{2ct} [(1-t)^2 c \eta \sigma^2 + 2c\bar{w} + t^2 - 1]$$

$$\begin{aligned}
g_s &= -\frac{2(1-t)^2}{c} + \frac{(1-t)^3(1-c\eta\sigma^2)}{2c(1-s+st)^3} (2-6s+6s^2-6ts^2-2s^3+4ts^3-2t^2s^3) + \frac{2\bar{w}(1-t)^2}{(1-s+st)^3} \\
&= \frac{(1-t)^2}{2c(1-s+st)^3} [2(1-t)^3(1+c\eta\sigma^2)s^3 - 6(1-t)^2(1+c\eta\sigma^2)s^2 \\
&\quad + 6(1-t)(1+c\eta\sigma^2)s + 2(1-t)(1-c\eta\sigma^2) + 4c\bar{w}t - 4] \\
&= \frac{(1-t)^2}{c(1-s+st)^3} \{ (1+c\eta\sigma^2)[(1-t)s-1]^3 + tc\eta\sigma^2 + 2c\bar{w}t - t \}
\end{aligned}$$

Within the interval of (0,1), $\frac{(1-t)^2}{c(1-s+st)^3}$ is positive. Let

$A = (1+c\eta\sigma^2)[(1-t)s-1]^3 + tc\eta\sigma^2 + 2c\bar{w}t - t$, A is monotonically increasing in s .

$$A(s=0) = (1-t)(1-c\eta\sigma^2) + 2c\bar{w}t - 2$$

$$A(s=1) = t[(1-t)^2c\eta\sigma^2 + 2c\bar{w} - t^2 - 1]$$

If $A(s=0) \geq 0$, i.e. $(1-t)(1-c\eta\sigma^2) + 2c\bar{w}t - 2 \geq 0$, $A > 0$ for all $s \in (0,1)$. If $A > 0$, g is increasing in s . Since $g(s=0)$ is positive, there is no solution for equation (1).

If $A(s=1) \leq 0$, i.e. $(1-t)^2c\eta\sigma^2 + 2c\bar{w} - t^2 - 1 \leq 0$, $A < 0$ for all $s \in (0,1)$. If $A < 0$, g is decreasing in s . When $g(s=1) > 0$, i.e. $(1-t)^2c\eta\sigma^2 + 2c\bar{w} + t^2 - 1 > 0$, there is no solution. When $g(s=1) \leq 0$, i.e. $(1-t)^2c\eta\sigma^2 + 2c\bar{w} + t^2 - 1 \leq 0$, there exists one solution.

If $A(s=0) < 0, A(s=1) > 0$, i.e.

$(1-t)(1-c\eta\sigma^2) + 2c\bar{w}t - 2 < 0, (1-t)^2c\eta\sigma^2 + 2c\bar{w} - t^2 - 1 > 0$, g is firstly decreasing and then increasing within the interval of (0,1). The solution of $A=0$ is

$$s = \frac{1 - \left[\frac{tc\eta\sigma^2 + 2c\bar{w}t - t}{1 + c\eta\sigma^2} \right]^{\frac{1}{3}}}{1-t}. \text{ Thus, if the value of } g \text{ at this point is positive, there is}$$

no solution for equation (1); if the value is 0, there is one solution, which is

$$\text{just } s = \frac{1 - \left[\frac{tc\eta\sigma^2 + 2c\bar{w}t - t}{1 + c\eta\sigma^2} \right]^{\frac{1}{3}}}{1 - t};$$

if the value is negative, there are two solutions for

equation (1), however, we need to take second-order condition into consideration as well: A is negative at the smaller point so that g_s (S.O.C) is negative, which means it gives maximum value, while at the larger point, g_s is positive, meaning it is a solution for minimum value, which should be omitted.

To sum up, there is one solution for equation (1) if one of the following conditions is satisfied: i). $(1-t)^2 c\eta\sigma^2 + 2c\bar{w}t - t^2 - 1 < 0$;

ii). $(1-t)(1 - c\eta\sigma^2) + 2c\bar{w}t - 2 < 0$, $(1-t)^2 c\eta\sigma^2 + 2c\bar{w}t - t^2 - 1 > 0$, and the value of

$$g \text{ at the point of } s = \frac{1 - \left[\frac{tc\eta\sigma^2 + 2c\bar{w}t - t}{1 + c\eta\sigma^2} \right]^{\frac{1}{3}}}{1 - t} \text{ is non-positive.}$$

3.3.2 Evasion case

If the firm allows evading, they have to solve a different problem and may offer a different contract. After a certain before-tax profit y is realized and observed by both parties, the manager might choose any $r \leq y$ to report as the firm's taxable income.

Assume tax authority conducts an investigation on all the firms with a probability of p , which is between zero and one; whenever an evading firm is investigated, it will be uncovered for certain. If an evading firm is detected, it has to pay the full tax plus a penalty; furthermore, the manager will be punished an extra amount of money.

Suppose the penalty follows a convex function, which means that the penalty will

become severer with the increase in the extent of evasion. For simplicity, we assume the penalty function for the firm to be $x(y-r)^2$ and for the manager to be $q(y-r)^2$, where $x > 0, q > 0$. When there is no evasion, both penalties are equal to zero.

The expected compensation the manager can earn for producing a profit y and reporting r , less than or equal to y , to the tax authority is

$$E(w) = b + p[s[(1-t)(y-b) - x(y-r)^2] - q(y-r)^2] + (1-p)s[y-b-t(r-b)]$$

where b is the fixed salary no matter tax evasion is detected or not; if with probability p , evasion is detected, the after-tax profit left to all the shareholders will be less, and additionally the manager will be punished on the personal level; if the firm is lucky enough, only $(r-b)$ is taxed, which results in more income from stocks. The manager will choose an r to maximize the above expected compensation. Here we assume r can be less than zero, meaning that there is situation of reporting a loss. For simplicity, we further assume that if a firm reports loss, it will get some tax payback or tax exemption in next period from the tax authority, which is equivalent to the product of the amount of loss and the tax rate. Anyway, with this assumption, we ensure that the above expected compensation function to be continuous and differentiable.

$$\begin{aligned} \frac{dE(w)}{dr} &= psx \cdot 2(y-r) + pq \cdot 2(y-r) - (1-p)st = 0 \\ \Rightarrow y-r &= \frac{(1-p)st}{2(psx + pq)} \end{aligned}$$

Since parameters p, s, t, x, q are all greater than zero, and p is less than one,

$(y - r)$ is a positive constant, which means that whenever evasion is allowed, the manager will always select underreporting. Moreover, she will always underreport a fixed amount regardless of how much true profit is for any fixed auditing rate, tax rate, penalty intensity, and the incentive strength. It is easy to find that r is increasing in auditing frequency and penalty rate on the level of firm as well as on the manager, the economic explanation of which is quite straightforward. r is decreasing in tax rate and incentive factor because under a higher tax rate, evasion helps to save more taxes, thus it is beneficial for the manager to practice more evasion, and a higher proportion of stocks means more weight of the total compensation on the after-tax profit, providing a higher incentive for the manager to evade. In a word, the manager's expected compensation if evasion is allowed by the owner equals to

$$\begin{aligned}
E(w) &= b + s(1-t)(y-b) - (psx(y-r)^2 + pq(y-r)^2) + (1-p)st(y-r) \\
&= b + s(1-t)(y-b) - (psx + pq) \frac{(1-p)^2 s^2 t^2}{4(psx + pq)^2} + \frac{(1-p)^2 s^2 t^2}{2(psx + pq)} \\
&= b + s(1-t)(y-b) + \frac{(1-p)^2 s^2 t^2}{4(psx + pq)}
\end{aligned}$$

The principal's optimization problem can be stated as follows:

$$\max_{a,b,s} E(p(1-s)((1-t)(y-b) - x(y-r)^2) + (1-p)(1-s)(y-b - t(r-b)))$$

subject to

$$E(-e^{-\eta[E(w) - \frac{1}{2}ca^2]}) \geq u(w) \quad (\text{IR})$$

and

$$a \in \arg \max_a E(-e^{-\eta[E(w) - \frac{1}{2}ca^2]}) \quad (\text{IC})$$

We follow the similar steps as in the no evasion case and simplify the manager's optimization problem first.

$$\begin{aligned}
E(-e^{-\eta[E(w)-\frac{1}{2}ca^2]}) &= E(-e^{-\eta[b+s(1-t)(y-b)+\frac{(1-p)^2 s^2 t^2}{4(psx+pq)}-\frac{1}{2}ca^2]}) \\
&= E(-e^{-\eta[b+s(1-t)(a+\varepsilon-b)+\frac{(1-p)^2 s^2 t^2}{4(psx+pq)}-\frac{1}{2}ca^2]}) \\
&= (-e^{-\eta[b+s(1-t)(a-b)+\frac{(1-p)^2 s^2 t^2}{4(psx+pq)}-\frac{1}{2}ca^2]}) E(e^{-\eta s(1-t)\varepsilon}) \\
&= (-e^{-\eta[b+s(1-t)(a-b)+\frac{(1-p)^2 s^2 t^2}{4(psx+pq)}-\frac{1}{2}ca^2]}) (e^{\frac{\eta^2 s^2 (1-t)^2 \sigma^2}{2}}) \\
&= (-e^{-\eta[b+s(1-t)(a-b)+\frac{(1-p)^2 s^2 t^2}{4(psx+pq)}-\frac{1}{2}ca^2-\frac{\eta s^2 (1-t)^2 \sigma^2}{2}]) = (-e^{-\eta \hat{w}'(a)})
\end{aligned}$$

To maximize the above term is equivalent to maximize $\hat{w}'(a)$ by choosing a .

$$\max_a \hat{w}'(a) = b + s(1-t)(a-b) + \frac{(1-p)^2 s^2 t^2}{4(psx+pq)} - \frac{1}{2}ca^2 - \frac{\eta s^2 (1-t)^2 \sigma^2}{2}$$

The first order condition is

$$s(1-t) - ca = 0 \Rightarrow a = \frac{s(1-t)}{c}$$

which is identical as that in no evasion case.

Substitute a and $(y-r)$ into the principal's utility function, her problem becomes

$$\max_{b,s} (1-s)(1-t)(\frac{s(1-t)}{c} - b) - \frac{p(1-p)^2 t^2 x s^2 (1-s)}{4(psx+pq)^2} + \frac{(1-p)^2 t^2 s(1-s)}{2(psx+pq)}$$

subject to

$$b + s(1-t)[\frac{s(1-t)}{c} - b] + \frac{(1-p)^2 t^2 s^2}{4(psx+pq)} - \frac{s^2 (1-t)^2}{2c} - \frac{\eta s^2 (1-t)^2 \sigma^2}{2} = \bar{w}$$

Again, reorganize the constraint and obtain the expression of b

$$b = \left(\frac{s^2(1-t)^2}{2c} + \frac{(1-p)^2 t^2 s^2}{4(psx + pq)} - \frac{\eta s^2(1-t)^2 \sigma^2}{2} - \bar{w} \right) / (-(1-s+st))$$

Substitute it into principal's objective function and obtain first order condition for s as:

$$\begin{aligned} & \frac{(1-t)^2(1-2s)}{c} + (1-t)^3(1-c\eta\sigma^2) \frac{2s+ts^2-4s^2+2s^3-2ts^3}{2c(1-s+st)^2} + \frac{\bar{w}t(1-t)}{(1-s+st)^2} \\ & + \frac{d}{ds} \frac{(1-p)^2 t^2 s(1-s)(2q+xs-qs+qts)}{4p(1-s+st)(sx+q)^2} = 0 \end{aligned} \quad (2)$$

$$\begin{aligned} \text{where } & \frac{d}{ds} \frac{(1-p)^2 t^2 s(1-s)(2q+xs-qs+qts)}{4p(1-s+st)(sx+q)^2} \\ & = \frac{t^2(1-p)^2}{4p(1-s+st)^2(sx+q)^3} \{ (qxt - qxt^2 - 2q^2 + 4q^2t - 2q^2t^2 - x^2t)s^3 \\ & + [q^2(1-t)(6-t) - 3qxt]s^2 - 2q^2(3-t)s + 2q^2 \} \end{aligned}$$

Denote the solution to equation (2) as s^* , and substitute it into the expression of b and get b^* . (b^*, s^*) is the best contract for the owner if she allows evasion.

As we have already shown before, the manager's expected compensation if the owner allows evasion and she does underreport is

$$\begin{aligned} E(w) &= b^* + s^*(1-t)(y - b^*) + \frac{(1-p)^2 t^2 s^{*2}}{4(ps^*x + pq)} \\ &= s^*(1-t)y + (1-s^* + s^*t)b^* + \frac{(1-p)^2 t^2 s^{*2}}{4(ps^*x + pq)} \\ \text{where } b^* &= \left(\frac{s^{*2}(1-t)^2}{2c} + \frac{(1-p)^2 t^2 s^{*2}}{4(ps^*x + pq)} - \frac{\eta s^{*2}(1-t)^2 \sigma^2}{2} - \bar{w} \right) / (-(1-s^* + s^*t)). \end{aligned}$$

Recall that if evasion is not permissible, the manager's compensation is

$$\begin{aligned} w &= b^\circ + s^\circ(1-t)(y - b^\circ) = s^\circ(1-t)y + (1-s^\circ + s^\circ t)b^\circ \\ \text{where } b^\circ &= \left(\frac{s^{\circ 2}(1-t)^2}{2c} - \frac{\eta s^{\circ 2}(1-t)^2 \sigma^2}{2} - \bar{w} \right) / (-(1-s^\circ + s^\circ t)). \end{aligned}$$

We also find that the optimal effort level is in the same function of s , which means that whenever s^* and s° are equal, the effort level will be the same under evasion case and no evasion case. Let us examine the total compensation in these two cases when $s^* = s^\circ$.

$$\begin{aligned}
E(w) &= s^*(1-t)y + (1-s^* + s^*t)b^* + \frac{(1-p)^2 t^2 s^{*2}}{4(ps^*x + pq)} \\
&= s^\circ(1-t)y - \left(\frac{s^{*2}(1-t)^2}{c} + \frac{(1-p)^2 t^2 s^{*2}}{4(ps^*x + pq)} - \frac{\eta s^{*2}(1-t)^2 \sigma^2}{2} - \bar{w} \right) + \frac{(1-p)^2 t^2 s^{*2}}{4(ps^*x + pq)} \\
&= s^\circ(1-t)y - \left(\frac{s^{\circ 2}(1-t)^2}{c} - \frac{\eta s^{\circ 2}(1-t)^2 \sigma^2}{2} - \bar{w} \right) \\
&= s^\circ(1-t)y + (1-s^\circ + s^\circ t)b^\circ = w
\end{aligned}$$

This means that if $s^* = s^\circ$, the optimal contract under the evasion case provides the same expected total compensation as the second-best compensation scheme in no evasion case does, thus no efficiency loss is associated with evasion.

Now we need to check whether s^* will be equal to s° . Compare equation (1) with equation (2), there is one more term containing s in the second equation, while all else are just identical, suggesting that only under some strict conditions, the solution will be the same, but in most situations, they are different from each other. We will discuss this in detail later in section 4.

3.4 Model 2

In model 2, it is assumed that the principal cannot observe true profit generated by the manager. She faces the possibility that the manager may exaggerate her performance

in order to earn more bonuses. The contract consists of three parts: a fixed salary b , a part related to reported profit (for simplicity, it is assumed to be linear as mentioned before), and a stock proportion:

$$w = b + mr + n(1-t)(y - b - mr) \text{ }^{10}$$

where m is the parameter for bonus and n is the stock fraction. The first two terms are paid as wage, thus are subtracted when calculating the taxable income, while the last term is a measure of the compensation as being a shareholder. All other assumptions are the same as in model 1.

The manager has three strategies: report $r < y$; report $r = y$; report $r > y$. The choice is made after true profit y is realized. We show the expected total compensation under these situations.

3.4.1 To underreport

When the manager chooses to report $r < y$, the expected compensation is

$$E(w) = b + mr + p(n((1-t)(y - b - mr) - x(y - r)^2) - q(y - r)^2) \\ + (1-p)n((1-t)(y - b - mr) + t(y - r))$$

Substitute $r = y$ into the above expression

$$E(w) = b + my + n(1-t)(y - b - my),$$

which is the compensation for truthfully reporting, meaning that the expression is applicable for any $r \leq y$.

First order condition is

¹⁰ The reason why the term of mr should be here is the same as why b is here as explained before.

$$y - r = \frac{(1-p)nt - (1-n+nt)m}{2(pnx + pq)}.$$

Notice that it is a constant for any settled tax system and designed contract, but it is no longer always positive as in model 1.

When it is positive, i.e. $(1-p)nt - (1-n+nt)m > 0$, the manager will choose to evade a constant $\frac{(1-p)nt - (1-n+nt)m}{2(pnx + pq)}$ regardless of the level of y . In this case, reported profit r is increasing in p , x , q , m , and decreasing in t and n . The explanations are the same as in model 1. For m , it means that when the bonus part accounts more in the total compensation relative to the stock part, the manager is less willing to increase the value of stocks by giving up what she can earn as bonuses, thus she will report more.

When it is zero, i.e. $(1-p)nt - (1-n+nt)m = 0$, the manager's best choice is to report $r = y$.

When it is negative, i.e. $(1-p)nt - (1-n+nt)m < 0$, the first order condition indicates that the manager's best choice is to report an r greater than y . However, we have limited r to be less than or equal to y in this situation, so the solution should be the corner solution that $r = y$.

3.4.2 To over-report

When the manager chooses to over-report, the firm has to pay more taxes, but there is no penalty if the over-reporting is found, also there is no refund for the over-paid taxes. The expected compensation is

$$\begin{aligned}
E(w) &= b + mr + n(y - b - mr - t(r - b - mr)) \\
&= (1 - n + nt)b + ny + (m(1 - n) - nt(1 - m))r
\end{aligned}$$

Substitute $r = y$ into it and we can get

$$\begin{aligned}
E(w) &= (1 - n + nt)b + ny + (m(1 - n) - nt(1 - m))y \\
&= b + my + n(1 - t)(y - b - my)
\end{aligned}$$

which means this expression is also applicable to $r = y$.

When the coefficient of r is positive, i.e. $m(1 - n) - nt(1 - m) > 0$, the manager should choose r as high as possible to maximize $E(w)$. There is an important point here that when over-reporting, the manager has to make sure that the firm is able to afford the taxes it declares, i.e. taxes to be paid should not exceed the realized true profit less what pays to her as wage. To show this in the mathematical term,

$$t(r - b - mr) \leq y - b - mr \Rightarrow r \leq \frac{y - (1 - t)b}{m + t(1 - m)}.$$

Therefore, the manager will report $r = \frac{y - (1 - t)b}{m + t(1 - m)}$ in such a situation. When the true taxable income is positive, which is assumed in the very beginning of our analysis, this r is greater than y , meaning that it does satisfy the constraint for this situation. We show the proof as follows.

Suppose that the true taxable income is positive, we have

$$\begin{aligned}
y - b - my &> 0 \Rightarrow (1 - m)y > b \\
&\Rightarrow (1 - m)(1 - t)y > (1 - t)b \\
&\Rightarrow y - ty - m(1 - t)y > (1 - t)b \\
&\Rightarrow y - (1 - t)b > (m + t(1 - m))y \\
&\Rightarrow \frac{y - (1 - t)b}{m + t(1 - m)} > y
\end{aligned}$$

When the coefficient is negative, the manager should report r as low as possible, that is just y in this situation.

When the coefficient is zero, reporting anything possible results in a same outcome; we assume that if this is the case, the manager will just honestly report y .

To sum up, the manager's strategy for how to report the profits depends on the contract offered. She will report

$$r = y - \frac{(1-p)nt - (1-n+nt)m}{2(pnx + pq)} < y, \quad \text{if } m < \frac{(1-p)nt}{1-n+nt};$$

$$r = y, \quad \text{if } \frac{(1-p)nt}{1-n+nt} \leq m \leq \frac{nt}{1-n+nt};$$

$$r = \frac{y - (1-t)b}{m + t(1-m)} > y, \quad \text{if } m > \frac{nt}{1-n+nt}.$$

For the owner, her payoff is

$$(1-n)(1-t)(y-b-my) + \frac{(1-n)((1-p)nt - (1-n+nt)m)}{4p(nx+q)^2} (2(nx+q)(1-t) + x(1-n+nt)m + (nx+q)(1-p)t),$$

if the manager underreports;

$(1-n)(1-t)(y-b-my)$, if the manager truthfully reports;

0, if the manager over-reports.

Obviously, the owner wants to avoid over-reporting since she will get nothing in the end; furthermore, for any fixed combination of m and n , the payoff from underreporting is larger than that from truthfully reporting as the extra term of $\frac{(1-n)((1-p)nt - (1-n+nt)m)}{4p(nx+q)^2} (2(nx+q)(1-t) + x(1-n+nt)m + (nx+q)(1-p)t)$ is

positive. Thus, whenever possible, the owner prefers underreporting; in order to

induce evasion, the contract designed by the owner should satisfy $m < \frac{(1-p)nt}{1-n+nt}$.

If no evasion exists, the problem is

$$\max_{a,b,m,n} (1-n)(1-t)(y-b-my)$$

subject to

$$E(-e^{-\eta[(1-n+nt)b+(m+n(1-t)(1-m))y-\frac{1}{2}ca^2]}) \geq u(\bar{w}) \quad (\text{IR})$$

and

$$a \in \arg\max_a E(-e^{-\eta[(1-n+nt)b+(m+n(1-t)(1-m))y-\frac{1}{2}ca^2]}) \quad (\text{IC})$$

Denote the solution as $(b^\circ, m^\circ, n^\circ)$.

If it turns out that the evasion will happen and the manager chooses the optimal evading amount as shown above, the problem is

$$\max_{a,b,m,n} ((1-n)(1-t)(y-b-my) + \frac{(1-n)((1-p)nt - (1-n+nt)m)}{4p(nx+q)^2} (2(nx+q)(1-t) + x(1-n+nt)m + (nx+q)(1-p)t))$$

subject to

$$E(-e^{-\eta[(1-n+nt)b+(m+n(1-t)(1-m))y+\frac{((1-p)nt-(1-n+nt)m)^2}{4(pnx+pq)}-\frac{1}{2}ca^2]}) \geq u(\bar{w}) \quad (\text{IR})$$

and

$$a \in \arg\max_a E(-e^{-\eta[(1-n+nt)b+(m+n(1-t)(1-m))y+\frac{((1-p)nt-(1-n+nt)m)^2}{4(pnx+pq)}-\frac{1}{2}ca^2]}) \quad (\text{IC})$$

Denote the solution as (b^*, m^*, n^*) .

Follow the same methods used in model 1, we can derive that for both cases, the

optimal level is in the same function of m and n , which is $a = \frac{m+n(1-t)(1-m)}{c}$.

The first order conditions tell that in most cases, (b^*, m^*, n^*) is different from $(b^\circ, m^\circ, n^\circ)$, which means that a^* will be different from a° .

Nevertheless, when we solve for m and n , we have not consider the fact that not all the combinations of (m^*, n^*) will lead to evasion. We discuss it in next section.

4. Discussion of the Results

As stated above in model 1, we have shown that usually the best compensation function under evasion case is not the same as that under no evasion case, which results in an effort level chosen by the manager when evasion exists depart from that of no evasion. Since the effort level we find in the no evasion case is the optimal effort level, evasion brings efficiency loss.

Proposition 1. If the owner of a corporation determines to operate on tax evasion, the optimal contract has to be altered from the second-best compensation scheme and thus incur efficiency loss.

We would like to explain the meaning of “efficiency loss” here. s^* is derived from solving the optimization problem under evasion, so it does maximize the owner’s expected utility when deciding to evade. This maximized expected utility is a result of an effort level a^* exerted by the manager and the activity of evasion. Evasion benefits the owner in the sense that underreporting by some certain extent, the savings in taxes exceeds the expected penalty involved in evasion; however, by choosing effort level a^* , instead of a° , the before-tax profit generated in the no evasion case can no longer be achieved, the loss of which is what we call “efficiency loss”. In a word, there are two opposite forces influencing the maximum expected utility for the owner: a change in effort level caused by evasion harms the firm while evasion itself benefits the firm. Therefore, the maximum expected utility under evasion may be higher than that under no evasion so that the owner prefers to evade,

but she has to bear the loss in the before-tax profit level, which will not be affected in an individual evasion problem; or perhaps the negative effect of evasion exceeds the positive effect, so that the owner finds it is better to just truthfully report and orders the manager to do in such a way.

This finding is similar to Chen and Chu's conclusion, but we believe our assumptions are more reasonable. It is consistent with the practical data presented in introduction. Compared to an individual, owners of a firm will not evade tax just observing that the expected gain is positive. They are aware of the fact that the compensation schemes under evasion can no longer optimally inspire the manager, which results in lower before-tax profit, and the loss might outweigh the gain on tax evasion. Firm owners take account of two tradeoffs, risk of being caught and risk of losing internal efficiency, while individuals only care about the former. Therefore, a firm will require an expected gain high enough to offset the efficiency loss other than just being "positive". Accordingly, we expect firms to be more cautious than individuals when attempting to evade and they should exhibit a lower underreporting rate.

The same finding can be drawn in model 2 as well, since similarly, in most cases, the contract will not be the same under evasion and no evasion, thus leading to a problem of over-compensation or under-compensation, both will result in a before-tax profit less than maximum level, but more importantly, it shows that there is another source of efficiency loss in reality.

When we state the optimization problem for evasion, it is not taken into account

whether the solution of m and n can induce evasion or not. If the solution satisfies the condition that $m < \frac{(1-p)nt}{1-n+nt}$, it is indifferent with model 1. If this condition does not hold, the optimal contract for evasion actually will mislead the manager to truthfully report or even over-report. The owner must give up the best contract, choosing another one that satisfies the above-stated condition so as to successfully induce evasion, altering from the optimal contract for evasion to which brings distortion in effort level and loss in before-tax profits.

Proposition 2. If the owner of a corporation can only induce evasion, instead of ordering evasion, there may be additional efficiency loss.

We set up two models, intending to separate these two sources of efficiency loss. One is resulted from the fact that evasion will lead to too much effort or too less effort, both reduces before-tax profits; the other one is that because of asymmetrical information, the owner has restrictions in designing the optimal contract to maximize her utility. Just like in a standard principal-agent model, the principal chooses a so-called second-best compensation scheme because effort level is private information of the agent; here the principal might have to choose a “third-best” compensation scheme because even output level is not known by her. The latter source of efficiency loss reinforces the former one, further reducing the probability for a firm to benefit from evasion relative to an individual or privately-held business.

5. Conclusion

We build a simple model to analyze corporate income tax evasion, based on Chen and Chu's work. The basic difference between corporate income tax evasion and individual income tax evasion is that the former needs cooperation between the manager and the firm owner, while the latter one can be more easily accomplished. The contract between the principal and the agent will be distorted from the second-best, so that it affects manager's decision on how much effort to take and thus affects the final output of the firm. However, for individuals or private businesses, there is no such incentive inducing issue, which means that the effort chosen is separate from evasion decision, i.e. output level will not decrease when evasion takes place.

This fact helps to explain the differences in observed data. For corporations to evade there is one more consideration of tradeoffs than individuals, efficiency loss, referring to the reduction in before-tax profits. This efficiency loss reduces the possibility of a firm to benefit from evasion as well as the extent of benefits, therefore reducing the willingness to evade, compared to individual income tax evasion.

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